Calculus for Business   Test III   Name:___________________

Student ID:___________________

<u>Show</u> your work. An answer alone will get NO credit. Circle your answer.

An illegible answer will not be graded. Do <u>not</u> use calculators.

1. (15 pts.)Suppose that the demand equation for a commodity at price p dollars is
$$q = D(p) = 600 - p^2.$$

   a. Find the elasticity of demand for this commodity and determine if the demand is elastic, inelastic, or of unit elasticity when $p = \$\,10$.

$$\text{Elasticity} = -D'(p)\cdot\frac{p}{D(p)} = -(-2p)\cdot\frac{p}{600-p^2} = \frac{2p^2}{600-p^2} = \quad p=10$$

$$= \frac{2(10)^2}{600-10^2} = \frac{200}{500} = \boxed{\frac{2}{5}}$$

$\frac{2}{5} < 1$ so the demand is $\boxed{\text{inelastic}}$

   b. Interpret the results from part a. (When the price is p = \$10, should you raise or lower the price of the commodity to increase total revenue?)

Since demand is inelastic at $p = \$10$, one should $\boxed{\text{raise the price } p}$ to increase revenue.

   c. Find the price for unit elasticity.

$$\text{Elasticity} = \frac{2p^2}{600-p^2} = 1$$

$$2p^2 = 600 - p^2$$

$$3p^2 = 600$$

$$p^2 = 200$$

$$p = +\sqrt{200} = \boxed{\$\,14.14}$$

2.(12 pts.) Find the absolute maximum and absolute minimum (if any) of the function

(Show your work!)

$f(x) = \ln(x^2 + x + 1)$ over the interval $-1 \leq x \leq 1$.

$x^2+x+1=0 \to$ no real solutions $\to$ no asymptotes of $\ln(\dots)$

$f'(x) = \dfrac{2x+1}{x^2+x+1} = 0 \to x = -1/2$

Absolute max value = $\ln 3$ at x= 1

Critical point x	f(x)
$x=-1$	$\ln(1)=0$
$x=-1/2$	$\ln(3/4) = -0.288 \leftarrow$ min
$x=1$	$\ln(3) = 1.099 \leftarrow$ max

Absolute min value= $\ln\left(\frac{3}{4}\right)$ at x = $-\frac{1}{2}$

3.(15 pts.)Suppose $ 2,000 is invested at an annual interest rate of 6%.

a. Find the balance after 8 years if money is compounded quarterly

$$2000\left(1+\frac{0.06}{4}\right)^{4\times 8} = \boxed{\$\,3220.65}$$

b. How long it would take to reach $6,000, if interest is compounded continuously?

$$2000 \cdot e^{0.06t} = 6000$$
$$e^{0.06t} = 3$$
$$0.06t = \ln 3$$
$$t = \frac{\ln 3}{0.06} = \boxed{18.3 \text{ years}}$$

c. Find the effective rate, if interest is compounded semiannually.(Show your work)(Calculate)

$$\left(1+\frac{0.06}{2}\right)^{2\times 1} = 1.0609 = 1 + 0.0609$$

effective rate = $\boxed{6.09\%}$

4.(6 pts.) How much should be invested now at 4 % compounded semiannually in order to have \$5,000 eight years from now?

$$x\left(1+\frac{0.04}{2}\right)^{2\times 8}=5000$$

$$x\,(1.02)^{16}=5000$$

$$x=\frac{5000}{1.02^{16}}=\boxed{\$3642.23}$$

5.(16 pts.) Find the derivative of each function and simplify your answer.

a. $f(x) = (x^3-2)e^{(x^6+5x+3)}$

$$f'(x)= 3x^2\cdot e^{x^6+5x+3}+(x^3-2)e^{x^6+5x+3}\cdot(6x^5+5)=$$

$$= e^{x^6+5x+3}\cdot(3x^2+6x^8+5x^3-12x^5-10)=$$

$$=\boxed{e^{x^6+5x+3}\cdot(6x^8-12x^5+5x^3+3x^2-10)}$$

b. $g(x) = \ln 7 + e^{5x} - e^4 + \frac{8}{\sqrt[3]{x}} - 5$

$$g'(x)= 0+e^{5x}\cdot 5-0+8\cdot\left(-\tfrac{1}{3}\right)x^{-\frac{4}{3}}-0=$$

$$=\boxed{5e^{5x}-\frac{8}{3x^{4/3}}}$$

6.(12 pts.) A store manager has been selling lamps at the price of \$6 per unit, and at this price, consumers have been buying 3,000 units per month. The owner of the store wishes to raise the price and estimates that for each \$1 increase in price, 1,000 fewer units will be sold each month. Each unit costs the store \$4.

a. At what price should the store owner sell the lamps to generate the greatest possible profit?(Show your work)

p = price per unit

"for each \$1 increase in price, 1000 fewer units sold"

Demand: $D(p) = b - 1000p$

$D(6) = b - 1000.(6) = 3000$

$b = 9000$

$\Rightarrow D(p) = 9000 - 1000p$

Profit: $P(p) = p \cdot D(p) - \underset{\text{cost}}{C(p)} = p(9000 - 1000p) - 4p =$

$= -1000p^2 + 8996p$

Maximize $P(p)$ for $p \geq 0$:

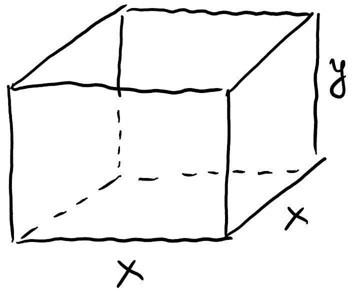
$P'(p) = -2000p + 8996 = 0 \rightarrow p = \frac{8996}{2000} = \boxed{\$4.498}$

$P''(p) = -2000 < 0 \Rightarrow$ we have a local max at

b.What is the maximum profit?

$P(4.498) = -1000(4.498)^2 + 8996(4.498) = \boxed{\$20232.00}$

7.(12pts.) You need to make an open box with a square base and volume of $64\ ft^3$. The material for the base of the box costs \$1 per square foot, and the material for the sides costs \$0.5 per square foot. Find the dimensions of the box that minimize the cost. (**Show your work.**)



Volume: $x \cdot x \cdot y = 64$

$\Rightarrow y = \frac{64}{x^2}$

Cost: $C = \$1 \cdot (\text{base area}) + \$0.5\,(\text{side area}) =$

$= 1 \cdot (x^2) + 0.5 \cdot (4xy) =$

$= x^2 + 2xy = x^2 + 2x \cdot \frac{64}{x^2} =$

$= x^2 + \frac{128}{x}, \quad x > 0$

Minimize $C(x)$:

$C'(x) = 2x - \frac{128}{x^2} = 0 \rightarrow x = (64)^{1/3} = 4$

$C''(x) = 2 + \frac{256}{x^3} > 0$ at $x = 4 \Rightarrow$ local $\underline{\text{min}}$

$\Rightarrow$ Cost is minimized when the square base has a side $\boxed{x = 4\ ft}$ and the box height is $y = 64/x^2 = 64/4^2 = \boxed{4\ ft}$.

8)(8 pts) Find the equation of the tangent line to $f(x) = 6e^{-2x}$ at x = 0

$f'(x) = 6 \cdot e^{-2x} \cdot (-2) = -12\, e^{-2x}$

$f'(0) = -12\, e^{0} = -12$

$f(0) = 6 \cdot e^{0} = 6$

Need line of slope -12 through point $(0,6)$:

$y - 6 = -12(x - 0)$

$\boxed{y = 6 - 12x}$

9. (8 pts) An economist has compiled these data on the gross domestic product (GDP) of a certain country:

Year	2000	2010
GDP (in billions)	100	180

Use this data to predict the GDP in the year 2020 if the GDP is growing exponentially, so that GDP = Ae^{kt}

year 2000 : $t=0$, $f(0) = 100$ billions
year 2010 : $t=10$, $f(10) = 180$ Billions
Find constants in $f(t) = Ae^{Kt}$:

$$f(0) = A \cdot e^{0} = A = 100 \rightarrow A = 100$$

$$f(10) = A \cdot e^{K \cdot 10} = 100\, e^{10K} = 180$$

$$e^{10K} = 1.8$$

$$K = \frac{\ln 1.8}{10}$$

Predict:
year 2020 : $t = 20$

$$f(20) = A \cdot e^{K \cdot 20} = 100 \cdot e^{\frac{\ln 1.8}{10} \cdot 20} = 100 \cdot e^{2 \ln 1.8} = \boxed{324 \text{ billions}}$$

10.(6 pts) Find all critical points of $g(x) = 5\, x^2 e^x$

$$g'(x) = 5 \cdot 2x \cdot e^{x} + 5x^2 \cdot e^{x} = (10x + 5x^2) \cdot e^{x} = 5x(2+x) \cdot e^{x} = 0$$

$x = 0$ $x = -2$

$$g''(x) = (10 + 10x)e^{x} + (10x + 5x^2)e^{x} = 5(x^2 + 4x + 2)e^{x}$$

Point $x = 0$:

$$g(0) = 5 \cdot 0^2 \cdot e^{0} = 0$$

$$g''(0) = 5(2)e^{0} = 10 > 0 \Rightarrow \boxed{(0,0) \text{ is local min}}$$

Point $x = -2$:

$$g(-2) = 5 \cdot (-2)^2 e^{-2} = 20/e^2$$

$$g''(-2) = 5(-2)e^{-2} = -10/e^2 < 0 \Rightarrow \boxed{\left(-2, \frac{20}{e^2}\right) \text{ is local max}}$$