Name XXXXXX

Problem Set #3 **SHOW ALL WORK- NO WORK= NO CREDIT**

All work is to be done individually, using only methods covered in this course, and with no additional resources.

1) Given the function: $f(x) = 2x^3 - 6x^2 + 8$.

Using Calculus techniques be sure to **find and state the coordinates of the relative extrema** and the **inflection point(s)** if they exist . Also, please state the coordinates of the **y-intercept**. Please **state the intervals where the function is increasing and decreasing** , along with **the intervals of concavity** , and use them to **draw an accurate sketch** of the function.

Please show your work in the space below and state your answers in the box to the right. **[21pts]**

Please draw your sketch on the grid on the **NEXT PAGE**. **[5pts]**

$f'(x) = 6x^2 - 12x = 6x(x-2) = 0 \rightarrow x = 0, 2$

$f'(x) > 0$ | $f'(x) < 0$ | $f'(x) > 0$ $\rightarrow x$

$f(x) \nearrow$ 0 $f(x) \searrow$ 2 $f(x) \nearrow$

Local max: $x = 0$

$f(0) = 2(0)^3 - 6(0)^2 + 8 = 8$ } point $(0,8)$

Local min: $x = 2$

$f(2) = 2(2)^3 - 6(2)^2 + 8 = 0$ } point $(2,0)$

$f''(x) = 12x - 12 = 12(x-1) = 0 \rightarrow x = 1$

$f''(x) < 0$ | $f''(x) > 0$ $\rightarrow x$

f(x) concave down 1 f(x) concave up

Inflection point: $x = 1$

$f(1) = 2(1)^3 - 6(1)^2 + 8 = 4$ } point $(1,4)$

y-intercept:

$x = 0$

$f(0) = 2(0)^3 - 6(0)^2 + 8 = 8$ } point $(0,8)$

Relative Extrema (identify each as a local max. or min.):

Local max: $(0,8)$

Local min: $(2,0)$

Inflection Points: $(1,4)$

y-intercept: $(0,8)$

Interval(s) f(x) is increasing:

$(-\infty, 0)$

$(2, \infty)$

Interval(s) f(x) is decreasing:

$(0,2)$

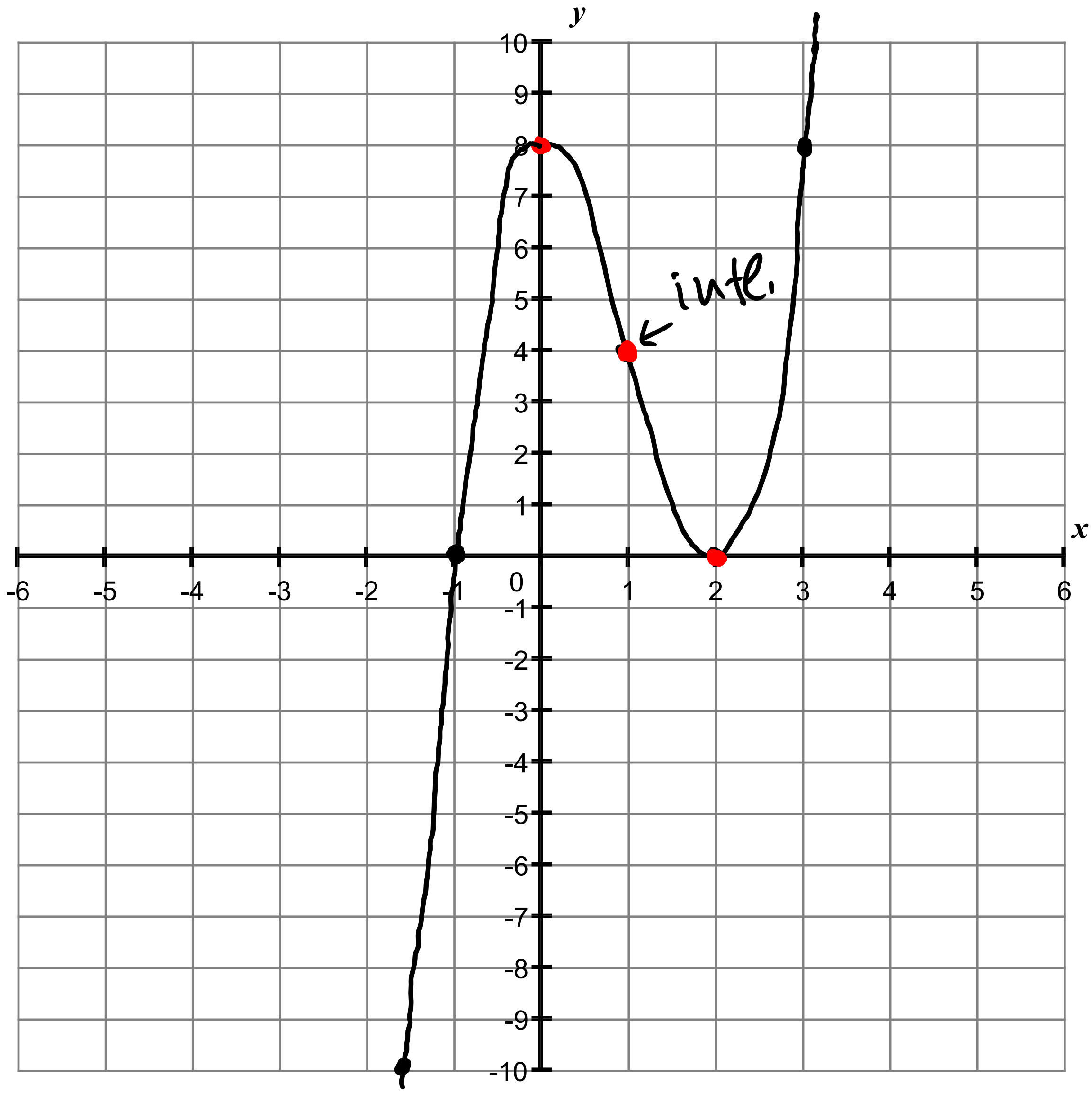
Interval(s) where f(x) is concave up:

$(1, \infty)$

Interval(s) where f(x) is concave down:

$(-\infty, 1)$

Grid for question #1:



2) Find the absolute maximum and absolute minimum values of each function below over the given intervals. Be sure to show all of the appropriate work to reach your conclusion. **[9pts]**

a) $f(x) = 2x^3 - 9x^2 + 80$, over the interval [-3,2].

f(x) is continuous over $[-3,2]$.

Critical values: outside interval

$f'(x) = 6x^2 - 18x = 6x(x-3) = 0 \rightarrow x = 0, 3$

Absolute max/min occur at boundaries or at critical values:

$f(-3) = 2(-3)^3 - 9(-3)^2 + 80 = -55$ ← Abs. min

$f(0) = 2(0)^3 - 9(0)^2 + 80 = 80$ ← Abs. max

$f(2) = 2(2)^3 - 9(2)^2 + 80 = 60$

Absolute max : $(0, 80)$

Absolute min : $(-3, -55)$

b) $g(x) = x + \dfrac{81}{x}$, over the interval $(0, \infty)$. **[11pts]**

vertical asymptote : $x = 0$

g(x) is differentiable over $(0,\infty)$ outside $(0,\infty)$

$g'(x) = 1 - \dfrac{81}{x^2} = 0 \rightarrow \dfrac{81}{x^2} = 1 \rightarrow x^2 = 81 \rightarrow x = -9, 9$

$g'(x) < 0$ $g'(x) > 0$ x

0 g(x) ↘ 9 g(x) ↗

Local min : $x = 9$

$g(9) = 9 + \dfrac{81}{9} = 18$ } point $(9, 18)$

Absolute min : $(9, 18)$ By Maximum-Minimum Principle 2

Absolute max : none

3) A watch manufacturer determines that in order to sell x units of a new watch, the price per unit in dollars must be: **[10pts]**

$$p(x) = 50 - .5x$$

The manufacturer also determines that the total cost of producing x units is given by:

$$C(x) = 10x + 3$$

a) Find the total revenue function, *R(x).*

$R(x) = x\,p(x) = x(50 - 0.5x) = -0.5x^2 + 50x$

b) Find the total profit function, *P(x*).

$p(x) = R(x) - C(x) = -0.5x^2 + 50x - \left(10x + 3\right) =$

$= -0.5x^2 + 40x - 3$

c) How many total units must the company produce and sell in order to maximize profits?

Maximize $p(x)$ for $x \geq 0$:

$p'(x) = -0.5(2x) + 40 = -x + 40 = 0 \rightarrow x = 40$

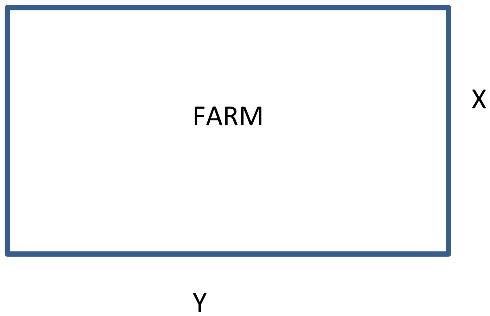
$p''(x) = -1 < 0 \rightarrow p(x)$ is concave down

$\Rightarrow$ Local and absolute max: $\boxed{x = 40 \text{ units}}$

d) What is the maximum profit?

$P(40) = -0.5(40)^2 + 40(40) - 3 = \boxed{\$797}$

4) A farmer is fencing in a rectangular area of his field for growing crops. He has 600 yards of fencing available to build the perimeter for all four sides. Using Calculus, determine the dimensions he should use to fence in the field if he wants to maximize the area? What is the maximum area? **[13pts]**



Perimeter = 600

$2x + 2y = 600$

$x + y = 300$

$y = 300 - x$

Domain: $\left.\begin{array}{l} x \geq 0 \\ y = 300 - x \geq 0 \rightarrow x \leq 300 \end{array}\right\} \; 0 \leq x \leq 300$

Area: $A(x) = x \cdot y = x(300 - x) = -x^2 + 300x$

Maximize $A(x)$ over $0 \leq x \leq 300$:

$A'(x) = -2x + 300 = 0 \rightarrow x = 150$

$A''(x) = -2 < 0 \rightarrow A(x)$ is concave down

Local and Absolute max at $\boxed{x = 150 \text{ yards}}$

$y = 300 - x = 300 - 150 = \boxed{150 \text{ yards}}$

Max area:

$A(150) = 150(300 - 150) = 150^2 = \boxed{22500 \text{ yards}^2}$

DIMENSIONS TO MAXIMIZE AREA:	$x = 150$ yards $y = 150$ yards	MAXIMUM AREA:	$22{,}500 \text{ yards}^2$

5) The half-life of Nickel-63 is 96 years. How old is an artifact that has lost 32% of its Nickel-63? **[8pts]**

$$k = \frac{\ln 2}{T} = \frac{\ln 2}{96}$$

$$N(t) = N_0 e^{-kt}$$

$$(1-0.32)\cancel{N_0} = \cancel{N_0} e^{-kt}$$

$$0.68 = e^{-kt}$$

$$\ln 0.68 = -kt$$

$$t = -\frac{\ln 0.68}{k} = -\frac{\ln 0.68}{\frac{\ln 2}{96}} = -96 \cdot \frac{\ln 0.68}{\ln 2} = \boxed{53.41 \text{ years}}$$

6) Evaluate and simplify the indefinite integrals below: **[16 pts]**

a) $\int \frac{1}{x^7}\, dx$

$$= \int x^{-7} dx =$$

$$= \frac{x^{-6}}{-6} + C = \boxed{-\frac{1}{6x^6} + C}$$

b) $\int 5x^{1/3}\, dx$

$$= 5\int x^{1/3} dx =$$

$$= 5 \cdot \frac{x^{4/3}}{4/3} + C =$$

$$= \boxed{\frac{15}{4} \cdot x^{4/3} + C}$$

c) $\int \left(x^9 + \frac{5}{\sqrt{x}} - 2e^{4x} \right) dx$

$$= \int x^9 dx + 5\int x^{-\frac{1}{2}} dx - 2\int e^{4x} dx =$$

$$= \frac{x^{10}}{10} + \frac{5x^{1/2}}{(1/2)} - 2 \cdot \frac{e^{4x}}{4} + C =$$

$$= \boxed{\frac{x^{10}}{10} + 10\sqrt{x} - \frac{1}{2} \cdot e^{4x} + C}$$

d) $\int \left(4e^{3x} + \frac{5}{x} - 6x^2 \right) dx$

$$= 4\int e^{3x} dx + 5\int \frac{dx}{x} - 6\int x^2 dx$$

$$= 4 \cdot \frac{e^{3x}}{3} + 5 \cdot \ln|x| - 6\frac{x^3}{3} + C$$

$$= \boxed{\frac{4}{3} \cdot e^{3x} + 5\ln|x| - 2x^3 + C}$$