1. (5 pts) Determine how many five-character codes can be formed if the first, second, and third characters are letters, the fourth character is a nonzero digit, the fifth character is an odd digit, and repetition of letters and digits are allowed. (A digit is 0, 1, 2, .., or 9.) **Show your work.** 1. _____

Multiply the possibilities for each character, 26 for the first/second/third letters, 9 for the fourth nonzero digit, 5 for the fifth odd digit: 26*26*26*9*5 = 790,920

A. 92
B. 3,510
C. 312,000
D. 790,920

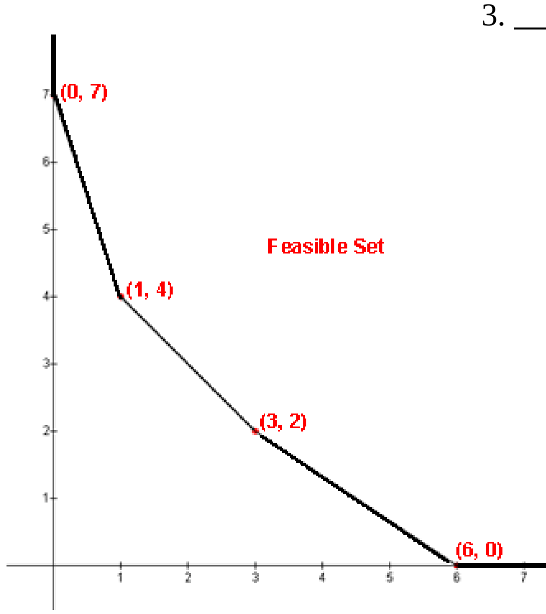
2. (5 pts) Suppose that a multiple choice exam has seven questions and each question has five choices. In how many ways can the exam be completed? **Show your work.** 2. _______

Multiply the possibilities for each question: 5*5*5*5*5*5*5 = 5^7 = 78,125

A. 35
B. 4,096
C. 16,807
D. 78,125

3. (5 pts) 3. ________

Given the feasible region shown to the right, find the values of x and y that **minimize** the objective function $7x + 8y$. **Show your work.**



A. There is no minimum.
B. $(x, y) = (6, 0)$
C. $(x, y) = (3, 2)$
D. $(x, y) = (1, 4)$
E. $(x, y) = (0, 7)$

Evaluate the objective function at each point:
for (0, 7) objective is 7(0) + 8(7) = 56
for (1, 4) objective is 7(1) + 8(4) = 39
for (3, 2) objective is 7(3) + 8(2) = 37
for (6, 0) objective is 7(6) + 8(0) = 42

The minimal value of the object function is 37 at point (3, 2).

4. (5 pts) Five customers in a grocery store are lining up at the check-out. In how many different orders can the customers line up? **<u>Show your work</u>.** 4. _______

There are 5 ways to select the first customer in the line, 4 ways to select the second, 3 ways to select the third etc. That gives 5*4*3*2*1 = 120 possible lineups.

A.	5
B.	25
C.	120
D.	3,125

5. (5 pts) A restaurant's menu has six appetizers, five entrees, and four beverages. To order dinner, a customer must choose one entrée and one beverage, and may choose one appetizer. (That is, a dinner must include one entrée and one beverage, but not necessarily an appetizer. An appetizer is optional.) How many different dinners can be ordered? **<u>Show your work</u>.**
5. _______

There are 5 choices for entrée, 4 choices for beverage, 7 choices for appetizers (including the no appetizer option) for a total of 5*4*7 = 140 possible dinners.

A.	140
B.	120
C.	24
D.	15

HINT: There are several ways to solve the problem. Here is something to think about, in general: Declining an item actually is a choice. For example, suppose that a person has an option of choosing a sweetener for a coffee drink. The person might use sugar or a particular sugar-free substitute, or decline a sweetener. Declining a sweetener means choosing no sweetener. (Choices are sugar, sugar-free, or none.)

6. (10 pts) A stamp collector has a set of five different stamps of different values and wants to take a picture of each possible subset of his collection (including the "empty set," depicting just the picture frame!), i.e., pictures showing no stamps, one stamp, two stamps, three stamps, four stamps, or five stamps. In each picture showing two or more stamps, the stamps are in a row. **<u>Showing your work</u>**, determine the maximum number of different pictures possible, when the difference between two pictures would be either in the number of stamps or in the horizontal order of the stamps. For example, if the stamp collector had just two different stamps (say A and B) of different values, he would have five pictures showing: A, B, AB, BA, and the empty frame.

Pictures with no stamps: 1
Pictures with 1 stamp: 5
Pictures with 2 stamps: 5*4 = 20 (5 choices for the first stamp, 4 choices for the second)
Pictures with 3 stamps: 5*4*3 = 60

Pictures with 4 stamps: 5*4*3*2 = 120
Pictures with 5 stamps: 5*4*3*2*1 = 120.

The total number of pictures possible is the sum of all cases: 1 + 5 + 20 + 60 + 120 + 120 = 326.

7. (12 pts) Let $U = \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$, $A = \{30, 50, 60, 90\}$ and $B = \{10, 20, 50, 80, 90\}$.
List the elements of the indicated sets. (No work/explanation required).

(a) $A \cap B$ = {50, 90}

(b) $A' \cap B$ = {10, 20, 80}
(Be sure to notice the complement symbol applied to *A*)

(c) $A \cup B'$ = {30, 40, 50, 60, 70, 90}
(Be sure to notice the complement symbol applied to *B*)

8. (9 pts) Use the given information to complete the following table.

n(U) = 80 , n(A) = 22, n(B′) = 35, n(A ∩ B) = 15. (No work/explanation required; note that U represents the universal set and the "primed" Sets are the complements.)

	A	**A′**	**Totals**
B	15	30	45
B′	7	28	35
Totals	22	58	80

9. (12 pts) 200 baseball fans in a Maryland county have been surveyed about the baseball teams they watch on TV. 103 fans watch the Washington Nationals. 90 fans watch the Baltimore Orioles. 170 watch the Washington Nationals or the Baltimore Orioles (or both).

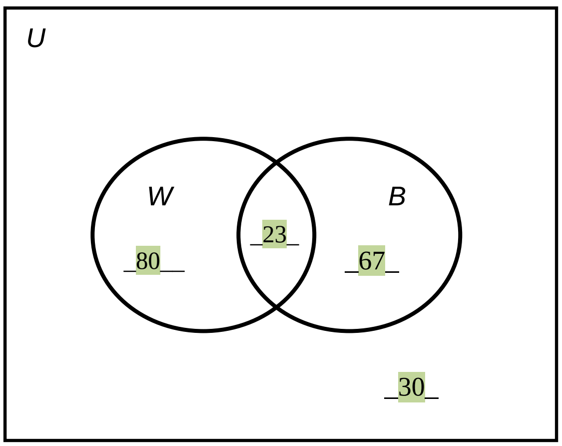
(a) How many of the fans watch both the Washington Nationals and the Baltimore Orioles? **Show work.**

W = 103, B = 90, WorB = 170.

WorB = W + B – WandB
Therefore the ones that watch W and B are:
WandB = W + B – WorB = 103 + 90 – 170 = 23.

(b) How many of the fans watch the Baltimore Orioles but not the Washington Nationals? **Show work.**
BbutnotW = B – WandB = 90 – 23 = 67.

(c) Complete the following Venn diagram, filling in the number of fans belonging in each of the four regions. Circle W = {fans who watch the Washington Nationals} and Circle B = {fans who watch the Baltimore Orioles}. (no explanation required)



10. (12 pts) A panel of 7 politicians is to be chosen from a group of 15 politicians.

(a) In how many ways can the panel be chosen? **Show work/explanation.**

Ordering inside the panel does not matter for an outcome so this is a combination problem:
15 choose 7 = ${}_{15}C_7$ = 6435 ways.

(b) Now suppose that the group of politicians consists of 5 Democrats, 7 Republicans, and 3 Independents. In how many ways can the 7-person panel be chosen if it must consist of 3 Democrats, 3 Republicans, and 1 Independent? **Show some work/explanation.**

Ways to select 3 Democrats out 5: ${}_5C_3 = 10$
Ways to select 3 Republicans out of 7: ${}_7C_3 = 35$
Ways to select 1 Independent out of 3: 3

The total number of ways is the product: 10*35*3 = 1050 ways.

11. (20 pts) Two kinds of cargo, A and B, are to be shipped by a truck. Each crate of cargo A is 25 cubic feet in volume and weighs 100 pounds, whereas each crate of cargo B is 40 cubic feet in volume and weighs 120 pounds. The shipping company collects $180 per crate for cargo A and $220 per crate for cargo B. The truck has a maximum load limit of 1,200 cubic feet and 4,200 pounds. The shipping company would like to earn the highest revenue possible.

(a) Fill in the chart below as appropriate.

	Cargo A (per crate)	Cargo B (per crate)	Truck Load Limit
Volume	25 cubic feet	40 cubic feet	1200 cubic feet
Weight	100 pounds	120 pounds	4200 pounds
Revenue	$180	$220	

Let x be the number of crates of cargo A and y the number of crates of cargo B shipped by one truck.

(b) State an expression for the total revenue R earned from shipping x crates of cargo A and y crates of cargo B.

$R = 180x + 220y$

(c) Using the chart in (a), state two inequalities that x and y must satisfy because of the truck's load limits.

$25x + 40y \leq 1200$ cubic feet
$100x + 120y \leq 4200$ pounds

(d) State two inequalities that x and y must satisfy because they cannot be negative.

$x \geq 0$
$y \geq 0$

(e) State the linear programming problem which corresponds to the situation described. Be sure to indicate whether you have a **maximization** problem or a **minimization** problem, and state the objective function and all the inequalities. (This part is mostly a summary of the previous parts)

Maximize $R = 180x + 220y$ subject to the constraints:
$25x + 40y \leq 1200$
$100x + 120y \leq 4200$
$x \geq 0$
$y \geq 0$

(f) Solve the linear programming problem. You will need to find the feasible region and determine the corner points. You do ***not*** have to submit your graph, and you do ***not*** have to show algebraic work in finding the corner points, but you must list the corner points of the feasible region and the corresponding values of the objective function.

Corner Point (*x*, *y*)	**Value of Objective Function**
(0, 0)	0
(0, 30)	6600
(42, 0)	7560
(24, 15)	7620

The maximum R = \$ 7,620 is achieved for x = 24 and y = 15.

(g) Write your conclusion with regard to the word problem. State **how many crates of cargo A and how many crates of cargo B** should be shipped in the truck, in order to earn the highest total revenue possible. State the value of that **maximum revenue**.

The maximum revenue is R = \$ 7,620.
It is achieved by shipping 24 crates of cargo A and 15 crates of cargo B.