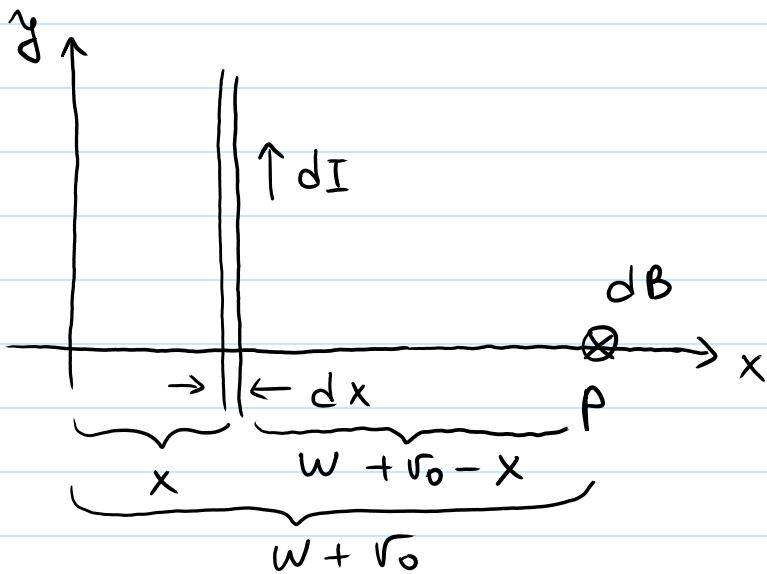


(7)



$$\frac{dI}{I} = \frac{dx}{w} \rightarrow dI = \frac{I}{w} dx$$

$$dB = \frac{\mu_0}{2\pi(w + r_0 - x)} \cdot dI = \frac{\mu_0 I}{2\pi w} \cdot \frac{dx}{w + r_0 - x}$$

$$B = \int_{x=0}^{x=w} dB = \frac{\mu_0 I}{2\pi w} \int_0^w \frac{dx}{w + r_0 - x} =$$

$$= \frac{\mu_0 I}{2\pi w} \left( - \ln|w + r_0 - x| \Big|_0^w \right) =$$

$$= \frac{\mu_0 I}{2\pi w} \left( \ln(w + r_0) - \ln(r_0) \right) =$$

$$= \frac{\mu_0 I}{2\pi w} \ln\left(\frac{w + r_0}{r_0}\right) = \boxed{\frac{\mu_0 I}{2\pi w} \ln\left(1 + \frac{w}{r_0}\right)}$$

(8)

$$E_x = -\frac{dV}{dx} = \text{-slope of } V(x)$$

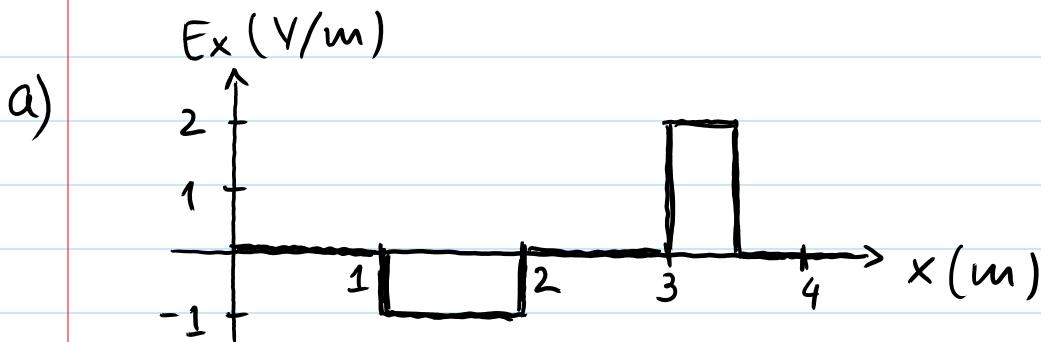
$$0 < x < 1 : \text{slope} = 0 \rightarrow E_x = 0$$

$$1 < x < 2 : \text{slope} = \frac{\Delta V}{\Delta x} = \frac{1V}{1m} = 1V/m \rightarrow E_x = -1V/m$$

$$2 < x < 3 : \text{slope} = 0 \rightarrow E_x = 0$$

$$3 < x < 3.5 : \text{slope} = \frac{\Delta V}{\Delta x} = \frac{-1V}{0.5m} = -2V/m \rightarrow E_x = +2V/m$$

$$x > 3.5 : \text{slope} = 0 \rightarrow E_x = 0$$



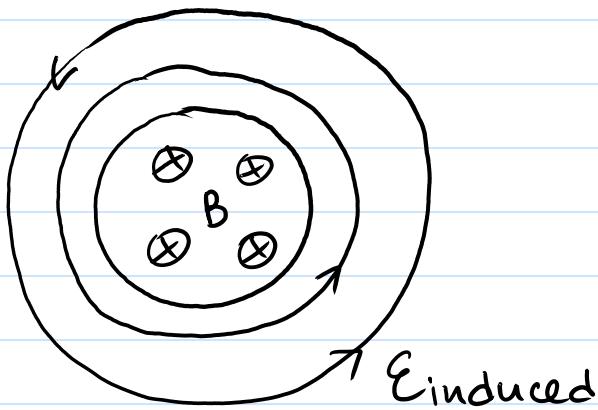
b) at  $x = 1.5 : E_x = -1V/m$

$$F_x = q \cdot E_x \Rightarrow F_x \text{ is negative}$$

$\uparrow$  negative  
positive

$\Rightarrow$  The positive charge will accelerate in [negative x direction].

9



Faraday's Law:

$$E_{\text{induced}} = - \frac{d\phi_B}{dt} = - \frac{d}{dt} \left( (-B) \cdot A \right) =$$

into page (negative flux)  
magnitude of  $\vec{B}$

$$= + A \cdot \frac{dB}{dt} = 1 \text{ m}^2 \cdot 1 \frac{\text{I}}{\text{s}} = + 1 \text{ V}$$

(counter clockwise)

a) work of person to move 1C clockwise

$F_{\text{electric}} = q \cdot E_{\text{induced}}$

$F_{\text{person}} \cdot ds$

$|F_{\text{person}}| = |F_{\text{electric}}|$

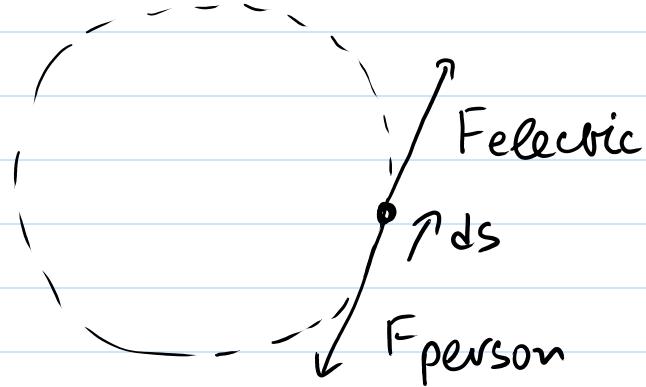
$$W = \int \vec{F}_{\text{person}} \cdot \overset{\rightarrow}{ds} = \int +|F_{\text{person}}| |ds| =$$

displacement

$$= \int |F_{\text{electric}}| \cdot |ds| = + |E_{\text{ind}} \cdot q| =$$

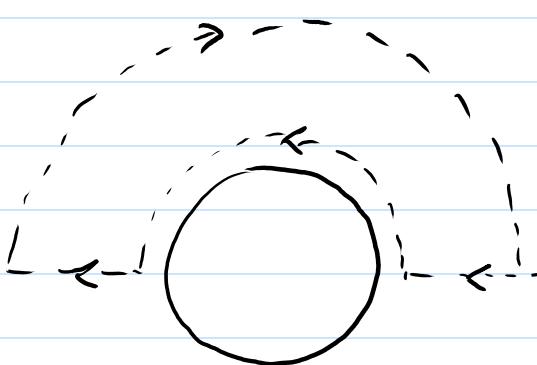
$$= + |1 \text{ V} \cdot 1 \text{ C}| = \boxed{+ 1 \text{ J}}$$

(c) moving the charge in opposite direction



$$W = \int \vec{F}_{\text{person}} \cdot d\vec{s} = \int -|F_{\text{person}}| \cdot |ds| = \boxed{-1 \text{ J}}$$

d)



Work of the induced electric field on the moved charge:

$$W_{\text{electric}} = q \cdot E_{\text{ind}} = q \cdot \left( -\frac{d\phi_B}{dt} \right)$$

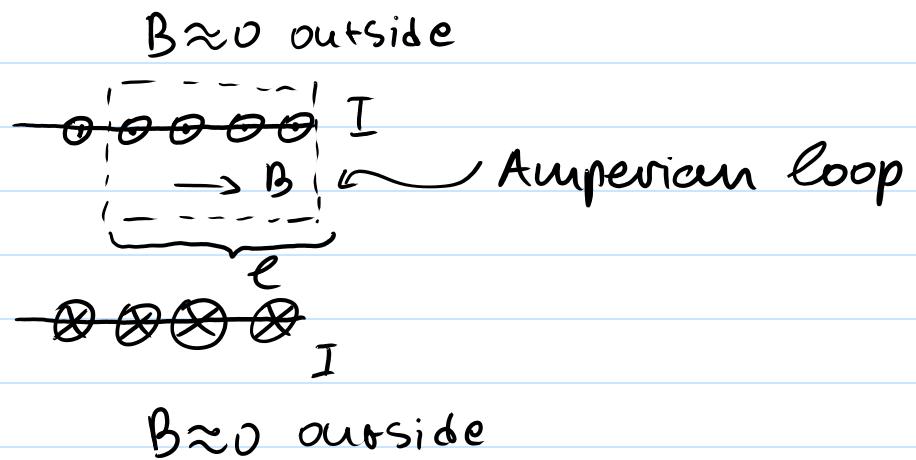
The magnetic field and its flux through the dashed path are zero:  $\phi_B = 0$

$$\Rightarrow \frac{d\phi_B}{dt} = 0 \Rightarrow W_{\text{electric}} = 0$$

$$W_{\text{person}} = -W_{\text{electric}} = -0 = \boxed{0 \text{ J}}$$

10

a) end view



Ampere's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$$

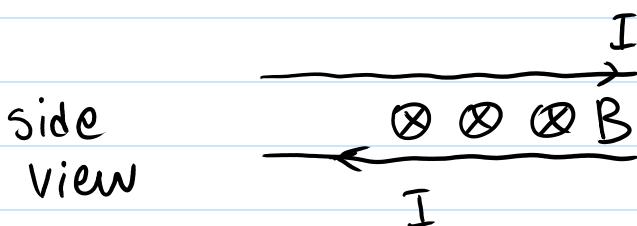
$$B \cdot l + 0 \cdot l + B \cdot 0 + B \cdot 0 = \mu_0 \cdot \frac{I}{w} \cdot l$$

↑      ↗  
 upper      left and right  
 side      sides:  $\vec{B} \perp d\vec{s}$  there

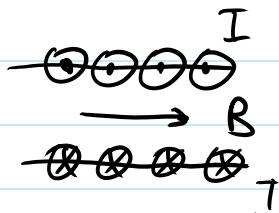
$$\Rightarrow B = \frac{\mu_0 I}{w}$$

(approximately homogeneous  
for  $d \ll w$ )

Direction:

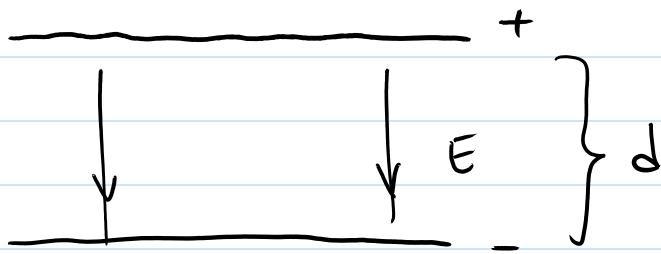


end view



(B is in the page)

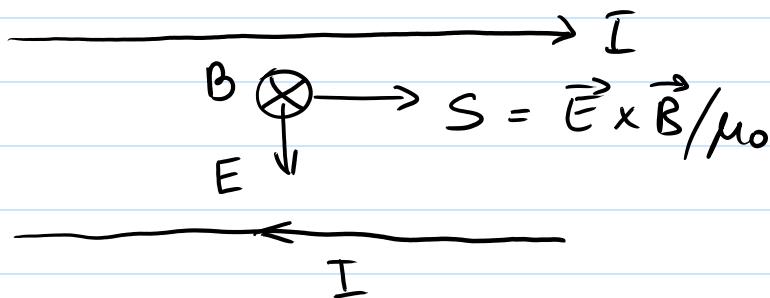
b)



$$E = \left| \frac{\Delta V}{\Delta x} \right| = \boxed{\frac{\Delta V}{d}} \quad (\text{direction from upper to lower strip})$$

c)

side view



$$S = E \cdot \frac{B}{\mu_0} \sin 90^\circ = \frac{\Delta V}{d} \cdot \frac{I}{W} \cdot 1 = \boxed{\frac{\Delta V \cdot I}{d \cdot W}}$$

Direction of  $S$  is determined by right-hand rule:

[to the right] on the side view.

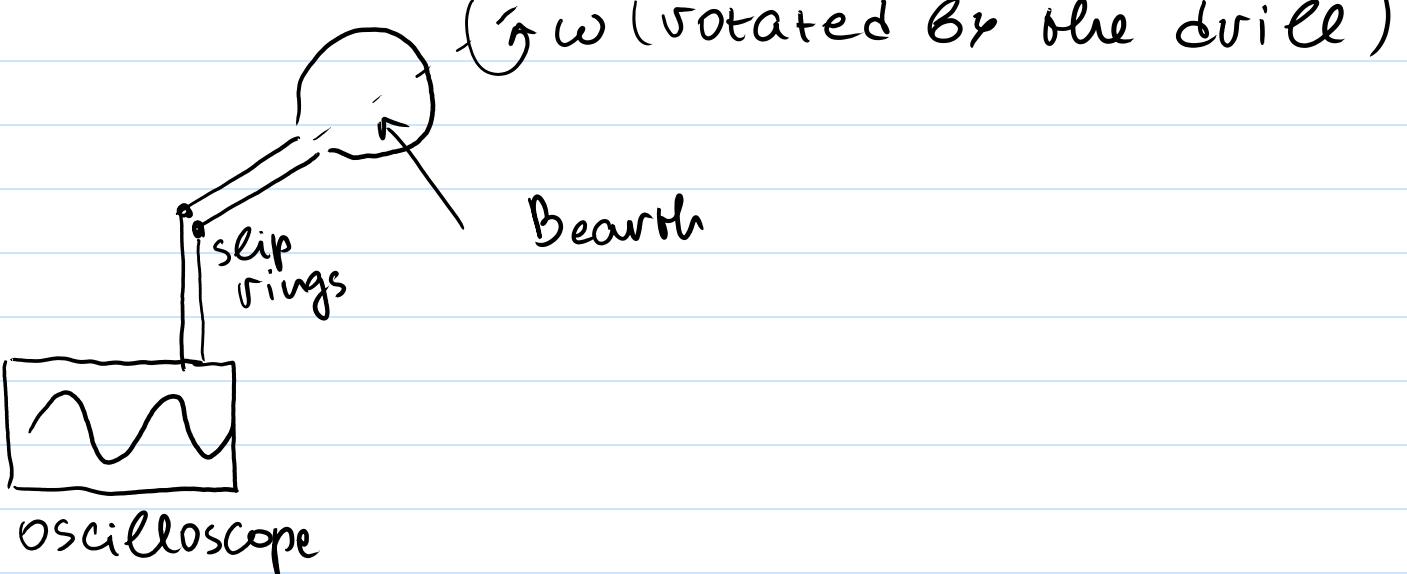
d) Energy  $= S \cdot (\text{perpendicular area}) = S \cdot d \cdot W =$

$$= \frac{\Delta V \cdot I}{d \cdot W} \cdot d \cdot W =$$

$$= \boxed{\Delta V \cdot I}$$

which is exactly the electric power delivered.

- (a) To measure the Earth's magnetic field  $B$ :
- 1) attach the coil to drill to make it rotate
  - 2) the magnetic field  $B$  will induce oscillating EMF in the rotating coil.
  - 3) orient the coil axis so that the amplitude of the induced EMF is maximal.  
(observed on oscilloscope).  
This is achieved when the coil rotating axis is perpendicular to  $\vec{B}$ .
  - 4) The magnitude of  $\vec{B}$  is proportional to the amplitude of the induced EMF observed on oscilloscope :



b) The EMF induced in the coil is due to the changing magnetic flux through the coil due to its rotation (Faraday's law).

I will measure the angular frequency  $\omega$  of the EMF and its magnitude  $E_{\max}$ .

c) I would need the coil diameter to determine its area ( $A = \pi d^2/4$ ).

d) Faraday's law:

$$E_{\text{ind}} = - \frac{d\phi_B}{dt} = - \frac{d}{dt} (\vec{B} \cdot \vec{A}) = - \frac{d}{dt} (B \cdot \frac{\pi d^2}{4} \cos(\omega t))$$

$$= B \cdot \frac{\pi d^2}{4} \cdot \omega \cdot \sin(\omega t)$$

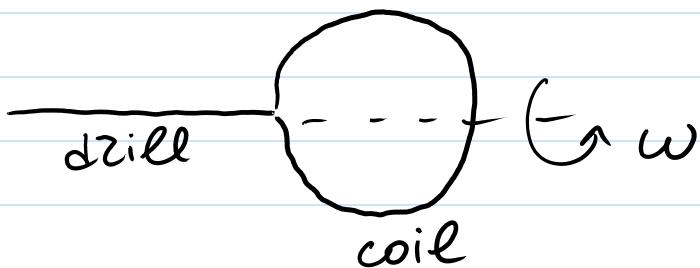
amplitude  $E_{\max}$

$$\Rightarrow E_{\max} = B \cdot \frac{\pi d^2}{4} \cdot \omega$$

$$\Rightarrow B = \boxed{\frac{E_{\max}}{\left(\frac{\pi d^2}{4}\right) \cdot \omega}}$$

( $\vec{B}$  must be  $\perp$  to rotation axis of coil)

e) use "lollipop" :



so that the coil rotation changes the angle between its plane and  $\vec{B}$ .

f) If the rotation axis is along  $\vec{B}$  :  
 $\Phi_B = 0$  and the  $E_{ind} = 0$  (zero signal).

That tells us that the component of  $\vec{B}$  along the rot. axis is not measured.

The signal is maximal when the flux  $\Phi_B = \vec{B}, \vec{A}$  has maximal amplitude i.e.

when the coil rot. axis is perpendicular to  $\vec{B}$ .

So, I will orient the rot. axis until I get maximal amplitude of induced EMF.

Then  $\vec{B}_{earth}$  is perpendicular to the rot. axis.

(12)

a)

$$X_C = R$$

$$\frac{1}{\omega_c C} = R \rightarrow \omega_c = \frac{1}{RC} = \frac{1}{(1\Omega)(10^{-4} F)} = \boxed{10^4 \frac{\text{rad}}{\text{s}}}$$

b) total impedance:  $Z = \sqrt{R^2 + X_C^2}$

$$V_{CO} = X_C \cdot \frac{V_o}{Z} = \frac{X_C}{\sqrt{R^2 + X_C^2}} V_o = \frac{V_o}{\sqrt{\left(\frac{R}{X_C}\right)^2 + 1}} =$$

$$= \frac{V_o}{\sqrt{1 + (RC\omega)^2}} = \frac{V_o}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

At high frequencies:  $\omega \gg \omega_c \rightarrow \frac{\omega}{\omega_c} \gg 1$

$$\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2} \approx \frac{\omega}{\omega_c}$$

$$\Rightarrow V_{CO} \approx \frac{V_o}{\left(\frac{\omega}{\omega_c}\right)} = \frac{V_o \omega_c}{\omega} = \boxed{\frac{V_o}{RC\omega}}$$

c) At low frequencies:  $\omega \ll \omega_c \rightarrow \frac{\omega}{\omega_c} \ll 1$

$$V_{CO} = \frac{V_0}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} = V_0 \left(1 + \left(\frac{\omega}{\omega_c}\right)^2\right)^{-1/2}$$

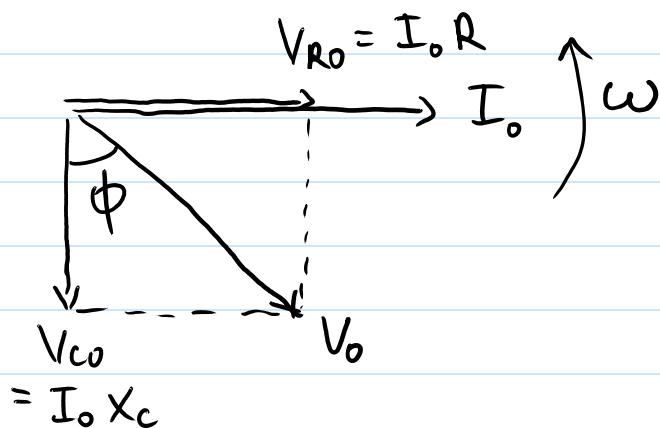
$$\approx \boxed{V_0 \left(1 - \frac{1}{2} \left(\frac{\omega}{\omega_c}\right)^2\right)}$$

d) at  $\omega = \omega_c$ :

$$V_{CO}(\omega_c) = \frac{V_0}{\sqrt{1 + \left(\frac{\omega_c}{\omega_c}\right)^2}} = \frac{V_0}{\sqrt{2}} = \boxed{\frac{1}{\sqrt{2}} \text{ Volts}} =$$

$$= 0.71 \text{ Volts}$$

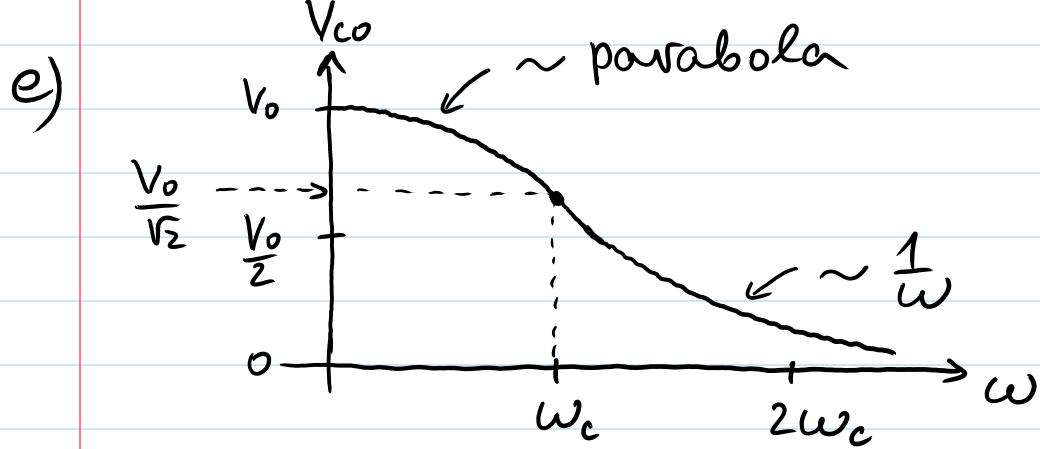
Phasor diagram:



The voltage across capacitor  $V_{CO}$  lags the total voltage  $V_0$  by  $\phi$ :

$$\tan \phi = \frac{V_{R0}}{V_{CO}} = \frac{R}{X_c} = 1 \quad (R = X_c \text{ when } \omega = \omega_c)$$

$$\Rightarrow \phi = \tan^{-1}(1) = \boxed{45^\circ} = \frac{\pi}{4} \text{ rad}$$



$$V_0 = 1 \text{ Volt}$$

$$\omega_c = 10^4 \text{ rad/s}$$

f) Resonance in RLC circuit:

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC} \rightarrow \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10^{-4} \text{ H})(10^4 \text{ F})}} = \boxed{\frac{1 \text{ rad}}{\text{s}}}$$

6)  $V_{CO} = X_L I_0 = X_L \frac{V_0}{Z} = \frac{X_L}{\sqrt{R^2 + (X_L - X_C)^2}}, V_0 =$

$\sim 0$  at resonance

$$= \frac{X_L}{R} \cdot V_0 = \frac{\omega_0 L}{R} \cdot V_0 = \frac{(1 \text{ rad/s})(10^{-4} \text{ H})}{1 \Omega} \cdot (1 \text{ V}) =$$

$$= \boxed{10^{-4} \text{ V}}$$