1. **Showing your work**,

(a) (4 points) When subject to an annual simple interest rate of 2%, how long will it take a present value (principal) to "grow" by 50%?

50% / 2% = 25 years

(b) (2 points) Does the "growth" in Part (a) depend on the principal? Support your answer by a brief explanation or computation.

The percent growth does not depend on the principal. For simple interest, it increases linearly with the time: (percent growth %) = (simple interest rate %)*(time in years)

(c) (4 points) What would be the growth (percent increase) in a principal if it were left in an account subject to an annual interest rate of 2% compounded annually for the length of time found in Part (a)?

After 25 years of annual compounding, the principal will be multiplied by a factor of $(1 + 0.02)^{25}$ = 1.64. That means the principal increased by 1.64 – 1 = 0.64 = 64%.

2. (25 points) Consider the following academic problem, involving four independent "savings accounts" A, B, C, and D:

At the end of August,

A. A deposit of $200.00 is made into a no-interest-bearing account (Account A) and also $200 is deposited into the account at the end of each following month for the rest of the year.
B. A one-time deposit of $800.00 is made into Account B, subject to an annual simple interest rate of 12%.
C. A one-time deposit of $800.00 is made into Account C, subject to an annual compound interest rate of 12%, compounded monthly.
D. A deposit of $200.00 is made into Account D and also $200 is deposited into the account at the end of each following month for the rest of the year, subject to an annual compound interest rate of 12%, compounded monthly.

Showing your work and rounding the answers to two decimal places,

(a) Populate the following table, to show the amount at the end of each month for each of the four accounts and the total interest earned at the end of November.

Amount (Future Value) at the end of each month after each end-of-the-month deposit

Account	End of August	End of September	End of October	End of November	Grand Total Interest Earned at the End of November
A	200	200+200= 400	400+200= 600	600+200= 800	800-800= 0
B	800	800*(1+0.12/12)= 808	800*(1+2*0.12/12)= 816	800*(1+3*0.12/12)= 824	824-800= 24
C	800	$800*(1+0.12/12)^1=$ 808	$800*(1+0.12/12)^2=$ 816.08	$800*(1+0.12/12)^3=$ 824.24	824.24-800= 24.24
D	200	$200*1.01^1+200=$ 402	$200*1.01^2+200*1.01^1+200=$ 606.02	$200*1.01^3+200*1.01^2+200*1.01^1+200=$ 812.08	812.08-800= 12.08

(b) Which of the above accounts would be a "Piggy Bank"? Support your answer briefly

It depends on the definition of "Piggy Bank" but account A has frequent deposits and no interest which is what a kid's "Piggy Bank" does.

(c) As a "Financial Analyst" in our MATH 106 class, what would you name or label Account D?

Account D has regular monthly deposits with compounded interest on each deposit, so I would name it annuity.

3. (15 points) When Matthew was born, his grandparents deposited $6000 into a special account for Matthew's college education. The account earned 7.3% interest compounded daily.

(a) How much will be in the account when Matthew is 18?

I am assuming 360 days in a year. The amount when Matthew is 18 is:
$6000*(1+0.073/360)^{18*360}$ = $22323.19

(b) If, upon becoming 18, Matthew arranged for the monthly interest to be sent to him, how much would he receive each 30-day month?

After additional 30 days of compounding the account will hold:
22323.19 * $(1+0.073/360)^{30}$ = \$ 22,459.39. The monthly interest that Matthew will receive each 30-day month is 22459.39 – 22323.19 = \$136.20

4. (15 points) A manufacturing company needs a piece of equipment to be replaced in 5 years at a cost of \$900,000.
To have this money available in 5 years, a sinking fund is established requiring making equal monthly payments (at the end of each month; no withdrawals) into an account paying 6.6% compounded monthly.

(a) How much should each payment be?

The monthly payment should be $\frac{900000\times(0.066/12)}{\left(1+\frac{0.066}{12}\right)^{5\times12}-1} = \$\ 12701.72.$

(b) How much interest is earned during the last year?

At the end of the 4th year, the sinking fund will hold a total of $12701.72 \times \frac{\left(1+\frac{0.066}{12}\right)^{4\times12}-1}{0.066/12} =$ $\$\ 695{,}561.53$. At the end of the 5th year, the sinking fund will hold \$900,000 by design.

The difference 900,000 – 695,561.53 = \$ 204,438.47 comes from the additional monthly payments during the last year and the additional interest earned during the last year.

The additional monthly payments during the last year are 12*12701.72 = \$ 152,420.64.

Therefore, the additional interest earned during the last year is:
204,438.47 – 152,420.64 = \$ 52,017.83.

5. (15 points)Kira buys a refrigerator for her big family for \$2,400 and agrees to pay for it in 18 equal monthly payments at an annual interest rate of 18% on the unpaid balance.

(a) How much are Kira's payments?

Her monthly payment is $\frac{2400\times(0.18/12)}{1-\left(1+\frac{0.18}{12}\right)^{-18}} = \$\ \mathbf{153.13}$
Her 18 monthly payments are 18*153.13 = \$ 2,756.34

(b) How much interest will she pay?

The interest she will pay is 2756.34 – 2400 = \$ 356.34

6. **A family purchased a home 10 years ago for $160,000, paying 20% down and signing a 30-year mortgage at 9% on the unpaid balance.**

(a) (10 points) How much is the unpaid loan balance after making 120 monthly payments?

The initial loan is 100%-20% = 80% of the price: 0.80*160000 = $128,000.

The monthly payment is $\frac{128000\times(0.09/12)}{1-\left(1+\frac{0.09}{12}\right)^{-30\times 12}}$ **=$1029.92**

The original loan after 120 months, compounded monthly at 9%, would be

$$\mathbf{128000}\times\left(\mathbf{1}+\frac{\mathbf{0.09}}{\mathbf{12}}\right)^{\mathbf{120}}=\$\,\mathbf{313,773.71}$$

The monthly payments after 120 months, also compounded monthly at 9%, would be

$$1029.92\times\frac{\left(1+\frac{0.09}{12}\right)^{120}-1}{0.09/12}=\$\,199{,}304.22$$

The remaining balance after 120 months would be

$$\mathbf{313773.71-199304.22}=\$\,\mathbf{114,469.49}$$

(b) **For Extra Credit (2 points): If equity in a home is defined as**

Equity = (current net market value) – (unpaid loan balance), find the equity in the family's home if its market value is $210,000.

equity = 210,000 – 114,469 = $ 95,531

7. (10 points) Effective yield, which is also called annual percentage yield [APY], effective interest rate, or true interest rate, is, in effect, the compounded interest on a one-dollar investment after one year.

Following the above interpretation of the effective yield, and invoking the compound interest formula, write the formula for the effective yield for each of the following cases and show your work. In other words, subtract 1 dollar from the future value of one dollar for each case, having been subjected to a compounding interest for one year.

Calculate the effective yield for the following interest rates:

(a) 4.93% compounded monthly

$\$1*(1 + 0.0493/12)^{12} - \$1 = 0.05043 = 5.043\%$

(b) 4.95% compounded daily

Assuming 360 days in a year: $\$1*(1 + 0.0495/360)^{360} - \$1 = 0.05074 = 5.074\%$

(c) 4.97% compounded quarterly

$\$1*(1 + 0.0497/4)^{4} - \$1 = 0.05063 = 5.063\%$

(d) 4.94% compounded continuously

$\$1*e^{0.0494*1} - \$1 = 0.05064 = 5.064\%$