

Phys 120

Problem Set 2

1

1st law:

A body at rest stays at rest unless acted on by an external force. A body in motion continues to travel with constant speed in a straight line unless acted on by an external force.

The law says that staying at rest or moving at constant velocity does not require an external force.

Example: a spaceship far away from the gravity of stars or planets is drifting at constant velocity with its engine turned off.

$$\underline{2^{\text{nd}} \text{ law:}} \quad \vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

The law says that the acceleration of a body is proportional to the net force acting on it and inversely proportional to its mass.

Example: A block of mass 1 kg acted upon by a force of 1 N, will have an acceleration:

$$a = \frac{1 \text{ N}}{1 \text{ kg}} = 1 \text{ m/s}^2$$

3rd law:

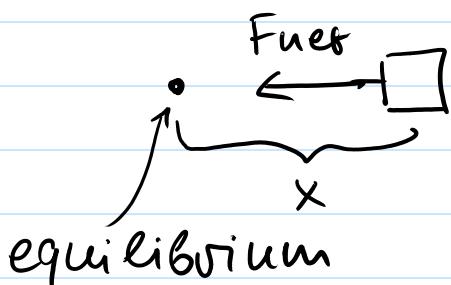
When two bodies A and B interact, the force from A on B, \vec{F}_{AB} , is equal in magnitude and opposite in direction to the force from B on A, \vec{F}_{BA} :

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

The law says that bodies interact with each other by force pairs: "action" and "reaction". If you know \vec{F}_{AB} , then you know \vec{F}_{BA} .

Example: a student pushes a wall with a force of 1 N. The wall pushes the student back by a force of 1 N.

- ② A simple harmonic motion is every motion for which the net force is proportional in magnitude to the displacement from an equilibrium position, and is directed towards the equilibrium position:



$$\vec{F}_{\text{net}} = -k \vec{x}$$

The same applies to the acceleration: $\vec{a} = -\frac{k}{m} \vec{x}$.

3 Power is energy change per unit time that is released, absorbed, emitted, utilized etc.

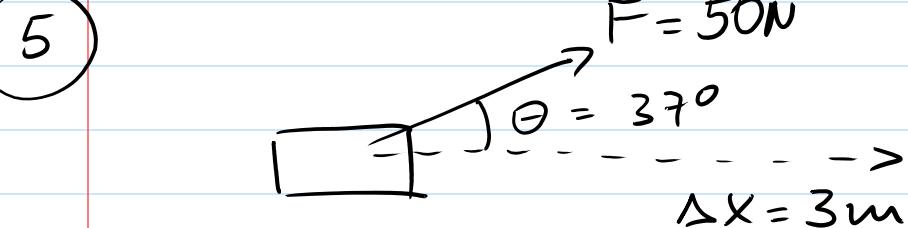
$$\text{power} = \frac{\Delta \text{Energy}}{\Delta \text{Time}}$$

4 Work is the change of energy due to the action of some force:

$$W = \Delta \text{Energy}$$

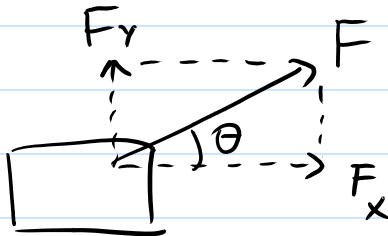
\Rightarrow Thus, power is the work done per unit time:

$$\text{Power} = \frac{\Delta \text{Energy}}{\Delta \text{Time}} = \frac{\text{Work}}{\Delta \text{Time}}$$



$$W = F \cdot \Delta x \cdot \cos \theta = (50\text{N})(3\text{m})(\cos 37^\circ) = [119.8 \text{ J}]$$

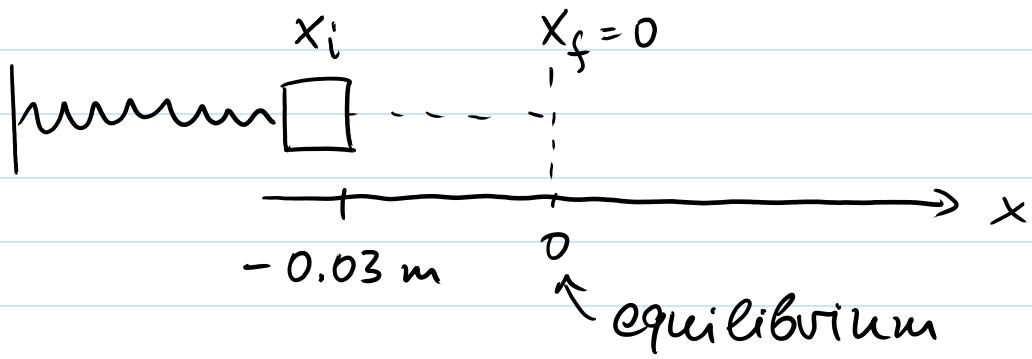
(6)



$$F_x = F \cos \theta = (50N) \cos 37^\circ = \boxed{39.9\text{ N}}$$

$$F_y = F \sin \theta = (50N) \sin 37^\circ = \boxed{30.1\text{ N}}$$

(7)



$$\begin{aligned}
 W_{\text{spring}} &= -\Delta PE_{\text{spring}} = -\left(\frac{kx_f^2}{2} - \frac{kx_i^2}{2}\right) \\
 &= \frac{k}{2}(x_i^2 - x_f^2) = \frac{(80\text{ N/m})}{2} \left((-0.03\text{ m})^2 - (0\text{ m})^2\right) = \\
 &= \boxed{0.036\text{ J}}
 \end{aligned}$$

\uparrow
 $3\text{ cm} = 0.03\text{ m}$

8

The kinetic energy of a body is the energy associated with its motion:

$$K = \frac{mv^2}{2}$$

where m is the mass of the body and $v = |\vec{v}|$ is its speed.

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Potential energy is the energy associated with the relative positions of different parts of the system.

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Total energy is the sum of kinetic and potential energies of all parts of the system.

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$$\begin{aligned} x(t) &= (4.0) \cos(\pi t + \frac{\pi}{4}) \\ &= A \cos(\omega t + \phi_0) \end{aligned}$$

what units?

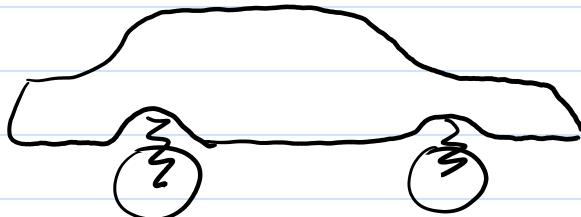
Amplitude: $A = \boxed{4.0}$

Angular frequency: $\omega = \boxed{\pi \text{ rad/s}}$

Frequency: $f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2} = 0.5 \text{ s}^{-1} = \boxed{0.5 \text{ Hz}}$

Period: $T = \frac{1}{f} = \frac{1}{0.5 \text{ s}^{-1}} = \boxed{2 \text{ s}}$

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Total mass :

$$M = \text{car} + \text{people} = 1300 \text{ kg} + 160 \text{ kg} = 1460 \text{ kg}$$

Total spring constant :

$$K = 4(20000 \frac{\text{N}}{\text{m}}) = 80000 \frac{\text{N}}{\text{m}}$$

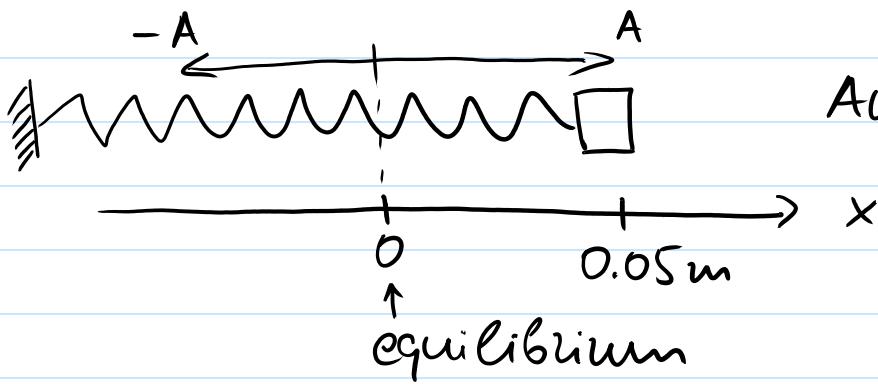
a) frequency :

$$f = \frac{1}{2\pi} \sqrt{\frac{K}{M}} = \frac{1}{2\pi} \cdot \sqrt{\frac{80000 \text{ N/m}}{1460 \text{ kg}}} = \boxed{1.18 \text{ Hz}}$$

b) period : $T = \frac{1}{f} = \frac{1}{1.18} = 0.85 \text{ s}$

Two vibrations : $2T = 2(0.85) = \boxed{1.7 \text{ s}}$

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Amplitude = 0.05m

a) Period:

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{(0.200 \text{ kg})}{5 \text{ N/m}}} = [1.257 \text{ s}]$$

b) Max speed: amplitude

$$v_{\max} = \omega A = \frac{2\pi}{T} \cdot A = \frac{2\pi}{1.257 \text{ s}} \cdot (0.05 \text{ m}) = [0.25 \text{ m/s}]$$

c) Max acceleration:

$$a_{\max} = \omega^2 A = \frac{kA}{m} = \frac{(5 \text{ N/m})(0.05 \text{ m})}{0.200 \text{ kg}} = [1.25 \text{ m/s}^2]$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5 \text{ N/m}}{0.200 \text{ kg}}} = 5 \text{ rad/s}$$

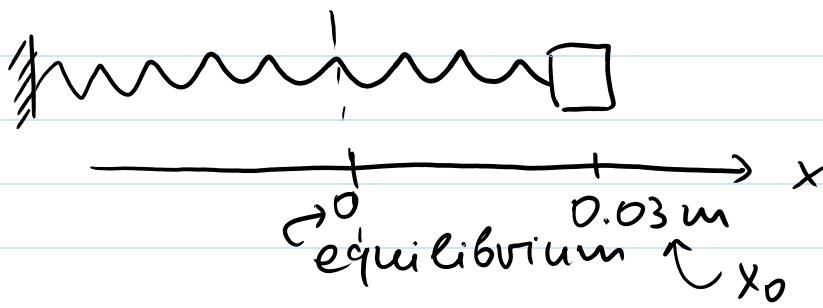
$x(t) = A \cos(\omega t)$ starts as A at $t=0$ and decreases

$$x(t) = [0.05 \cos(5t)]$$

$$v(t) = \frac{dx}{dt} = (0.05)(-5) \sin(5t) = [-0.25 \sin(5t)]$$

$$a(t) = \frac{dv}{dt} = (-0.25)(5) \cos(5t) = [-1.25 \cos(5t)]$$

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a) Total energy can be calculated at the initial point and stays constant:

$$E = KE_0 + PE_0 = \underbrace{\frac{mV_0^2}{2}}_0 + \underbrace{\frac{KX_0^2}{2}}_0 =$$

(starts at rest)

$$= \frac{KX_0^2}{2} = \frac{(20 \text{ N/m})}{2} (0.03 \text{ m})^2 = \boxed{0.009 \text{ J}}$$

The max speed is when the body is passing through equilibrium ($x = 0$):

$$E = \frac{mV_{\max}^2}{2} + \frac{K \cdot 0^2}{2}$$

$$E = \frac{mV_{\max}^2}{2}$$

$$V_{\max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.009 \text{ J})}{0.5 \text{ kg}}} = \boxed{0.19 \text{ m/s}}$$

b) Use conservation of energy:

$$E = \underbrace{\frac{mv_0^2}{2}}_0 + \frac{kx_0^2}{2} = \frac{mv^2}{2} + \frac{kx^2}{2}$$
$$kx_0^2 = mv^2 + kx^2$$

$$mv^2 = k(x_0^2 - x^2)$$

$$v = \pm \sqrt{\frac{k}{m}(x_0^2 - x^2)} = \pm \sqrt{\frac{20\text{N/m}}{0.5\text{kg}}((0.03\text{m})^2 - (0.02\text{m})^2)}$$
$$x_0 = 3\text{cm} \quad x = 2\text{cm}$$

$$= \pm 0.1414 \approx \boxed{\pm 0.14 \text{ m/s}}$$

moving to left OR right

c) $x = 2\text{ cm} = 0.02\text{ m}$

$$PE = \frac{kx^2}{2} = \frac{(20\text{N/m})}{2} \cdot (0.02)^2 = \boxed{0.004 \text{ J}}$$

$$KE = \frac{mv^2}{2} = \frac{0.5\text{kg}}{2} (0.14\text{m/s})^2 = \boxed{0.005 \text{ J}}$$

check:

$$\bar{E} = KE + PE = 0.004 + 0.005 = 0.009 \text{ J} \checkmark$$